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SUBLIMATION OF WIND-TRANSPORTED SNOW - A MODEL

by R. A. Schmidt, Jr.

ABSTRACT

Sublimation from blowing snow is estimated from the balance of heat and mass transfer on a volume of air and nonuniform ice particles. Size and the distribution of sizes are important particle factors, but fragmented and abraided shapes appear to increase sublimation only slightly above the estimate for spherical particles. Humidity and temperature are the overriding environmental factors, but atmospheric pressure and solar radiation are also important. The effect of turbulence on convection around the particles does not appear important in view of the small particle sizes encountered. Rather, turbulent transfer determines the snow concentration profile, and the gradients of heat and water vapor necessary to balance the sublimation process.

Keywords: Snow density, snow samplers, weather control, evaporation, sublimation.

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Sublimation of Wind-Transported Snow — A Model

by

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Contents

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Although a small process in the total hydrologic cycle, sublimation of solid precipitation during redistribution by wind may be a significant part of the water balance in certain regions. The objective of this work is to formulate a mathematical model to estimate the mass of snow that returns to vapor as the air-snow mixture moves downwind over a horizontal snow surface.[2]

Following a brief description of the phenomenon, sublimation of snow is considered in three parts. **Part I** uses an existing equation for sublimation of an ice sphere to evaluate the effects of meteorological factors, and a modification demonstrates the influence of particle shape. **Part II** shows that natural snow concentrations are not large enough to cause significant particle interaction, so that the results of Part I apply to a cloud of snow particles as well. **Part III** combines this knowledge with expressions for the vertical gradients of windspeed, snow concentration, temperature, and humidity, to estimate sublimation from a vertical column above a horizontal surface. Discussion of this model leads to several interesting consequences which should be grounds for fruitful experimentation.

The title employs "sublimation" to describe the process under discussion. Although sublimation can be used to denote either deposition or vaporization, the phase change from solid to vapor is of primary concern here. The process is assumed to be quasi-stationary, and sublimation estimates are instantaneous values that do not take into account the change in surface area of the particles as the process proceeds. Philip (1964) has shown that, while not fundamentally correct, such analysis provides useful approximations for many meteorological problems. Calculations are included both to provide more meaning to the equations and to given values that help keep the numerous factors in their proper perspective. In most cases, these results are presented graphically. The symbols are defined as they are introduced, and are listed in the appendix for reference.

The fact that the snow particles vaporize, either partly or sometimes completely, during transport by wind is not directly observable, since measurement problems are not yet solved, even for the relatively simple situation of transport across a horizontal surface. At windspeeds sufficient to move snow, natural airflow is turbulent, and windspeed increases with height. Snowflakes formed during precipitation are quickly reduced toward spherical shapes by mechanical abrasion, so that the windblown particles become like sand. As a particle leaves the snow surface to become suspended in the turbulent air stream, a boundary layer develops around it. If the vapor pressure of water in the surrounding atmosphere is different from the saturation value, a vapor pressure gradient forms across the boundary layer and sublimation begins. The rate of the process depends on how rapidly air is exchanged in the particle boundary layer. Energy, in the form of heat, is transferred to or from the particle surface by conduction, convection, and radiation. Water vapor is transferred by diffusion and convection. These same mechanisms operate in a cloud of snow particles, since the interparticle spacing far exceeds the depth of particle boundary layers under natural concentrations of snow suspended in turbulent air.

[2] *My initial interest in this problem was generated by discussions with R. D. Tabler. M. D. Hoover and M. Martinelli, Jr. helped at each stage of the development. J. L. Kovner and U. Radok reviewed the manuscript with great insight.*

PART I
SUBLIMATION OF A SINGLE PARTICLE

A. An Ice Sphere

Two laws of physics provide the starting point for theoretical considerations of the sublimation process. Conservation of mass provides a relationship in the form of the diffusion equation. The relationship for heat transfer comes from the law of conservation of energy.

1. Diffusion and Conduction

For mass transfer, the classical approach (Houghton 1933) is based on the differential equation relating the rate of loss of mass (dm/dt) of a spherical particle (radius r) to the surface area of the particle, the diffusivity (D) of water vapor in air, and the vapor pressure difference at the particle surface. The resulting equation for evaporation of a sphere in still air is

$$(dm/dt) = 4\pi Dr(\rho - \rho_r) \qquad [1]$$

where ρ is the water vapor density in the remote environment and ρ_r is the value at the particle surface. This equation expresses the rate of sublimation (or condensation) of a spherical particle by diffusion without ventilation.

Heat transfer by conduction to a sublimating spherical particle depends on the surface area, the temperature difference, and the thermal conductivity of air, K. The expression is

$$L_s\ (dm/dt) = 4\pi Kr(T_r - T) \qquad [2]$$

where L_s is the latent heat of sublimation. Temperature in the remote environment is T, and T_r is the surface temperature of the particle.

2. Convection

When a spherical particle sublimates in a steady air stream, the rate is higher and mass transfer varies around the particle surface. Frössling (1938) predicted this pattern from boundary layer theory, and verified his calculations with measurements of mass transfer around a naphthalene bead. The rate was maximum on the exposed face of the sphere, and decreased to a minimum at the point where the boundary layer separated from the particle surface. A smaller maximum occurred in the "eddy" area on the downstream surface. Temperature differences measured by Ranz and Marshall (1952) show a similar pattern around a ventilated drop.

Both Frössling (1938) and Kinzer and Gunn (1951) introduce "wind factors" by which the righthand sides of [1] and [2] are multiplied to account for increased sublimation due to ventilation. These ventilation factors are related by boundary layer theory to the particle Nusselt number $(Nu) = 2rK/D$ for heat transfer, and to an analogous ratio for mass transfer, the Sherwood number (Sh), often denoted by (Nu') in the literature. Thus, the theoretical equations are

$$(dm/dt) = 2\pi Dr\ (\rho - \rho_r)\ (Sh) \qquad [3]$$

$$L_s(dm/dt) = 2\pi Kr(T_r - T)(Nu) \qquad [4]$$

where (Sh) and (Nu) must be determined by experiment. Theory predicts, however, that (Nu) is a function of Reynolds number $(Re) = 2rV/\nu$ and Prandtl number $(Pr) = C_p\mu/K$, while the analogous Sherwood number is a function of (Re) and the Schmidt number $(Sc) = \mu/\rho_a D$. In these dimensionless groups, V is the ventilation velocity, μ is the dynamic viscosity, ρ_a is the density, and C_p is specific heat, all for air in this case. The kinematic viscosity is $\nu = \mu/\rho_a$.

At this point, the Clausius-Clapeyron equation $(1/P_s)(dP_s/dT) = L_s M/RT^2$ may be used to approximate the water vapor density difference in [3], where P_s is the saturation vapor pressure, M denotes the molecular weight of water, and R is the universal gas constant. Thorpe and Mason (1966) show that [3] and [4] may be combined to give the following expression for sublimation rate of a spherical ice particle:

$$\frac{dm}{dt} = \frac{2\pi r\ (\rho/\rho_{sT} - 1)}{\frac{L_s}{KT}\ \frac{1}{(Nu)}\left(\frac{L_s M}{RT} - 1\right) + \frac{1}{D\rho_{sT}\ (Sh)}} \qquad [5]$$

where ρ_{sT} represents the saturation density of water vapor at temperature T. The quantity $(\rho/\rho_{sT} - 1)$ is the undersaturation of the environment with respect to water vapor, and, following their notation, is denoted by σ.

Results of their experimental work with ice spheres $(0.3\ \text{mm} \leq r \leq 1.8\ \text{mm})$ at ventilation speeds from 25 to 100 cm sec^{-1} show the Nusselt number and the Sherwood number are practically the same, and are proportional to the square root of the Reynolds number.

$$(Nu) \simeq (Sh) = 1.88 + 0.58\ (Re)^{1/2} \qquad [6]$$

where (Re) ranged between 10 and 200. Values

of D in their computations are about 7 percent lower than the International Critical Tables value, in keeping with the experience of other workers, for example, Ranz and Marshall (1952).

Mean ventilation velocity, V, is the average relative velocity of air with respect to the particle. In these experiments, particle position was fixed in a moving air stream of low turbulence intensity (1 to 3 percent). The wind factor, F, by which sublimation is increased due to convection, has a theoretical value of unity when the Reynolds number is zero. However, [6] provided the best fit to their experimental data, so that $F = 1/2\ (Nu)$ is slightly less than unity with no ventilation. Ventilation may increase the diffusion of water vapor from the ice sphere by as much as five times if particle Reynolds numbers as high as 200 are involved (fig. 1).

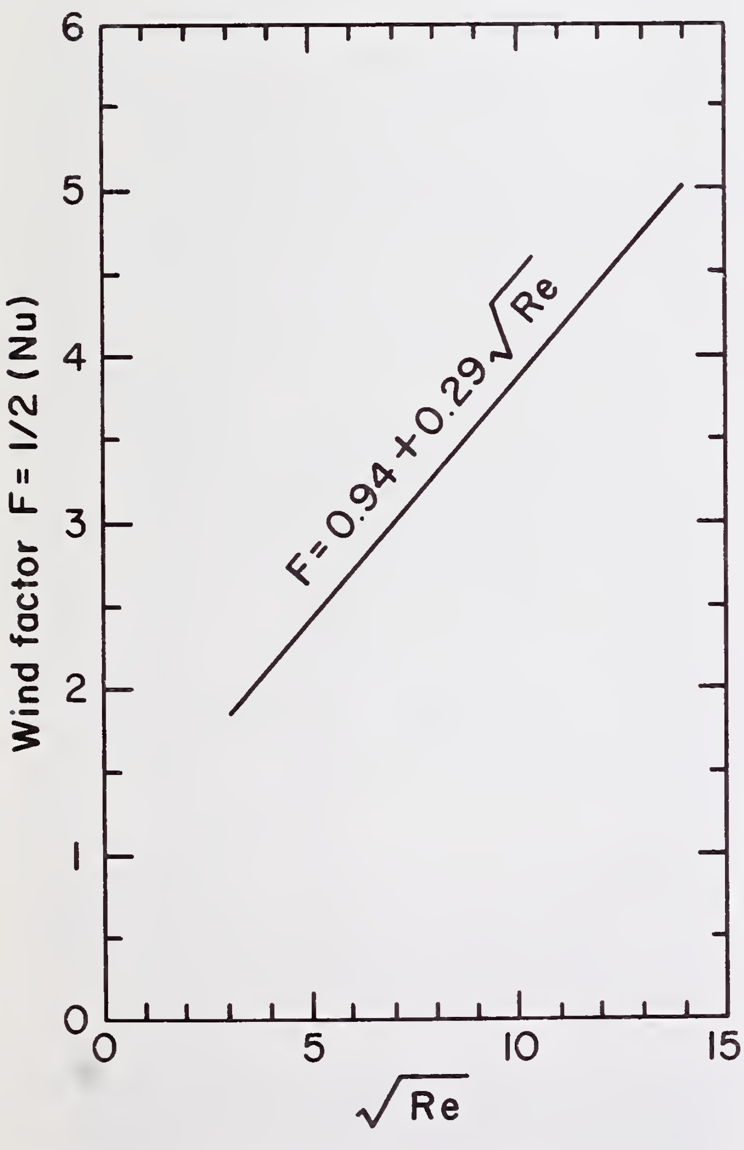


Figure 1.--Experiments by Thorpe and Mason (1966) show that ventilation increases sublimation by a factor (F) proportional to the square root of the Reynolds number, where (Re) = (2rV/ν) ranged from 10 to 200.

3. Particle Temperature

For water drops evaporating in still air, measurements by Johnson (1950) confirm the relation $(T_r - T) = (DL_s/K)\ (\rho - \rho_r)$. This expression for the temperature difference between the particle and its environment comes from assuming steady state equilibrium of heat and mass transfer expressed by [1] and [2]. Multiplying [1] by L_s gives an expression equivalent to [2]. In practice, the value of ρ_r is taken as the saturation vapor density at particle surface temperature T_r. The value of ρ, the ambient vapor density, is determined from the relative humidity of the environment and its temperature T.

Direct measurements by Kinzer and Gunn (1951) and Ranz and Marshall (1952) show that, for practical purposes, the surface temperature of a water drop evaporating with ventilation is equal to the wet bulb temperature estimated from psychrometric tables. Kinzer and Gunn (1951) also found the temperature of suddenly ventilated drops adjusted to the wet bulb temperature in only a few seconds.

For this model, particle temperatures are assumed to equal ice bulb temperatures. In the temperature range from -20° C. to 0° C., $(T_r - T)$, taken as the ice bulb depression, is less than 0.5° C. for relative humidities greater than 70 percent over the elevation range 0 to 4 km. Thorpe and Mason (1966) assumed the temperature difference $(T_r - T)$ was so small for conditions below freezing that the saturation vapor density at the particle surface was taken as that for ambient temperature T. Thus, particle temperature does not enter directly in [5].

4. Calculations

Together, [5] and [6] provide sublimation rates for ice spheres under conditions of moderate ventilation, at temperatures below freezing, without considering the radiation of heat to the particle. Some calculations will provide a graphical picture of the relation between factors. Before calculations can be made using [5], however, a number of "constants" must be assigned values. This has been done in the tables in the appendix.

Figure 2 shows the sublimation rate as a function of particle diameter for several ambient temperatures and mean ventilation velocities. All values were computed for 1000 mb atmospheric pressure, and 90 percent relative humidity. The sublimation rate at -10° C. is almost twice

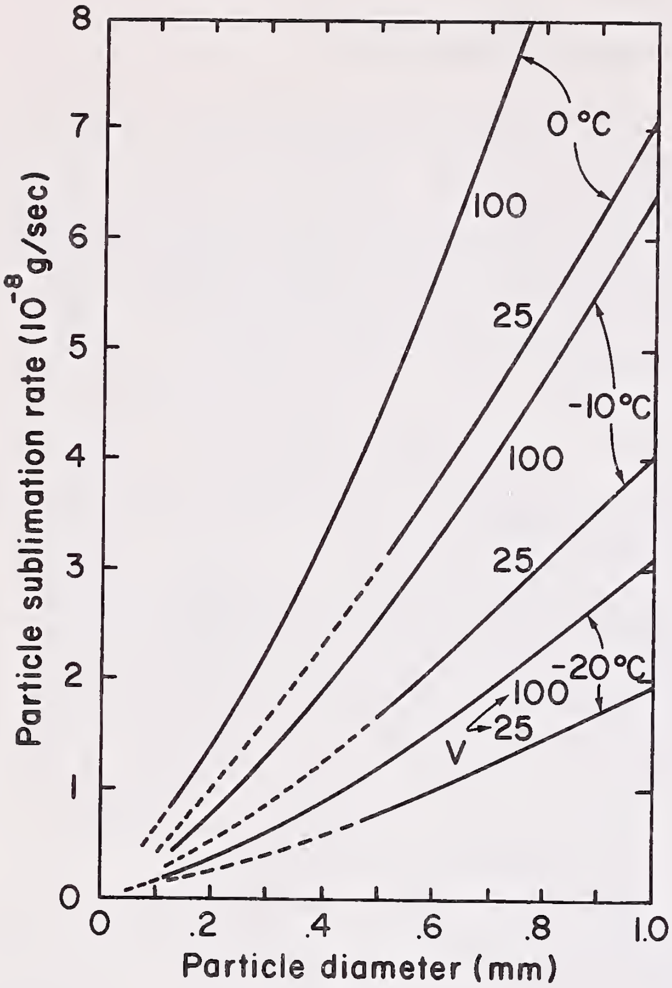


Figure 2.--The sublimation rate of a single ice sphere almost doubles for each 10-degree increase in ambient temperature between -20° C. and 0° C. Ventilation rate (V in cm/sec) has a smaller effect, according to equation [5] with values at 1000 mb. and 90 percent vapor density with respect to saturation over ice. (The dashed portion of the curves are extrapolations below (Re) = 10.)

the rate at -20° C. throughout the size range, and more than doubles when sphere diameter is doubled. Examination of [5] reveals that sublimation is directly proportional to the water vapor deficit in the surrounding environment. Doubling this undersaturation doubles the sublimation rate; 80 percent relative humidity would give twice the sublimation rates shown in figure 2.

Rate of sublimation decreases for smaller particles, but the loss becomes a larger percentage of particle mass. This follows from equations [5] and [6], which predict that sublimation rate varies as the 3/2 power of particle radius, and the fact that total particle mass varies with the third power of radius (fig. 3).

Particles studied by Thorpe and Mason were larger than most windblown snow. Budd, Dingle, and Radok (1966) reported mean particle diameters near 100 μ in the Antarctic, and Kojima (1969) presented particle size distributions with modes in the 20-40 μ diameter class, measured at Hokkaido University, Japan. Measurements on water drops by Kinzer and Gunn (1951) showed a rapid decrease in mass transfer for diameters below 150 μ when the relative velocity was equal to drop fall velocity. They point out that fog and cloud-size drops (around 25 μ) evaporate at about the rate predicted for a drop at rest. At low Reynolds numbers (0 to 7), the mechanism is not completely known, but air is apparently entrained by viscous forces and the drop tends to evaporate into an air shell around it. (Radius of curvature does not exert an important influence on vapor pressure, for particles larger than 1 micron.)

Perhaps the best limit to use in applying equation [5] to small particles is (Re) = 10, the lower limit given by Thorpe and Mason (1966) which agrees with the arguments of Kinzer and Gunn (1951). A 100 μ particle, ventilated by a 100 cm/sec air stream at -20° C., (Re = 8.6) is slightly outside this limit. With 90 percent relative humidity and 1000 mb atmospheric pressure, equation [5] predicts the sublimation rate will be $1.5 \cdot 10^{-9}$ g/sec.

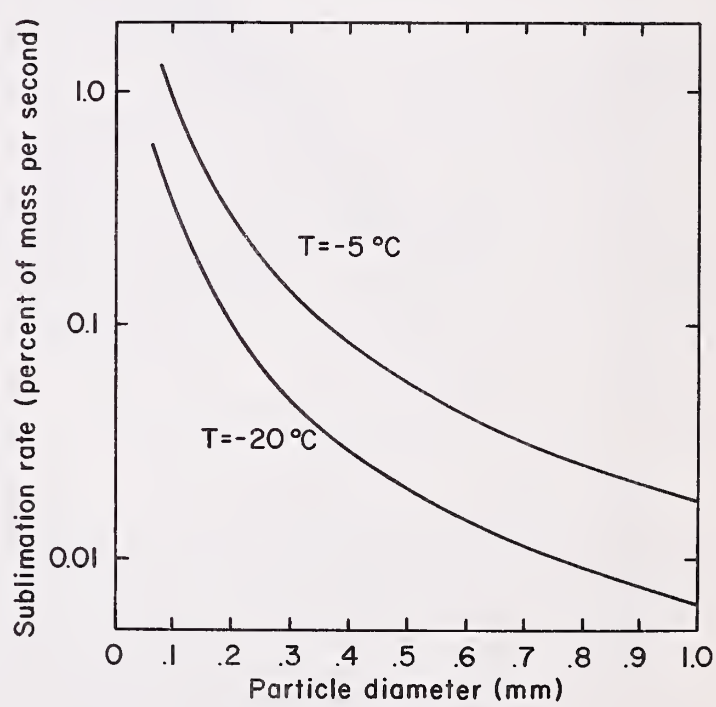


Figure 3.--The sublimation rate calculated by equation [5] represents a much larger percentage of the total mass for small particles. Calculations plotted here are for 1000 mb pressure and a vapor density that is 90 percent of the saturation value over ice. Particle density is assumed to be 0.92 g/cm³.

5. Radiation

For laboratory studies, most experimenters have justified neglecting heat transfer by radiation as part of the sublimation or evaporation process. However, Bergen [3] shows this factor is important for sublimation of a snowflake under solar radiation. Since redistribution of snow by wind may occur with clear skies, radiation will be included in this model. Following Bergen's procedure, a term is added to the heat transfer relation [2] so that

$$L_s(dm/dt) = 4\pi Kr(T_r - T) + Q \qquad [7]$$

where Q is the net rate at which heat is transferred to the particle by radiation. If the heat transfer expression [7], with the radiation term, is used in place of [2] to derive the equation for particle sublimation rate by the argument of Thorpe and Mason (1966), the result is

$$\frac{dm}{dt} = \frac{2\pi r\sigma(Nu) - \frac{Q}{KT}\left(\frac{L_s M}{RT} - 1\right)}{\frac{L_s}{KT}\left(\frac{L_s M}{RT} - 1\right) + \frac{1}{D\rho_{sT}}} \qquad [8]$$

To evaluate Q, some expression for the total radiation balance on the particle must be formulated. This will certainly be a widely varying term since conditions range from clear nights to clear days, with all cloudy and foggy situations between.

According to definitions outlined by van de Hulst (1957), a particle causes extinction of incident radiation equal to the amount it scatters and absorbs. Scattered radiation includes that defracted, refracted, and reflected by the particle. Of the incident radiation flux, S_o, the portion absorbed by the particle is equal to $C_{abs} S_o$, when C_{abs} is an area defined as the absorbing cross section of the particle. An absorbing efficiency may also be defined so that $E_{abs} = C_{abs}/G$ where G is the particle's geometric cross section ($G = \pi r^2$ for a sphere). If ζ is the particle albedo, $E_{abs} = 1 - \zeta$.

The total radiation flux intercepted by a spherical particle, suspended in turbulent air above a horizontal snow surface, will be taken as the sum of direct and scattered solar radiation. If S_o is the total solar radiation flux from above, and λS_o is that portion reflected by the horizontal snow surface, then the total radiation load on the particle is $S_o + \lambda S_o$, where the horizontal surface albedo, λ, may be different from the particle surface albedo.

[3] *Bergen, James D. An estimate of sublimation losses due to redeposition. Unpublished semiannual report of watershed management research, 1966. Rocky Mt. Forest and Range Exp. Stn., Fort Collins, Colo.*

The amount of this incident flux absorbed by the particle is $C_{abs}(S_o + \lambda S_o)$, or $E_{abs}G(S_o + \lambda S_o)$. It is equal to Q, the net rate of radiant heat transfer to the particle, and with $E_{abs} = 1 - \zeta$ and $G = \pi r^2$, for a spherical particle

$$Q = (1 - \zeta)(1 + \lambda)\pi r^2 S_o \qquad [9]$$

Values of albedo for a horizontal snow surface range from 75 percent or less for old snow, to almost 100 percent for new snow surfaces (Geiger 1965). A conservative value of $\lambda = 0.8$ has been used in calculations below. No estimates of snow particle albedo could be found in the literature, and a calculation based on particle optics was not formulated because no assumed surface characteristics seemed well founded. Instead, the value $\zeta = 0.5$ was taken as a very rough estimate.

This analysis neglects thermal radiation from the horizontal surface to the particle, because large differences between surface and particle temperatures do not seem likely under the natural conditions considered here. When snow is transported by wind just above asphalt, or a bare soil surface, thermal radiation might become an important additional heat flux to the particle.

Figure 4 compares sublimation rates computed from [8] and [9] with rates predicted by [5]. Most obvious is the fact that radiation contributes more in sublimation from large particles. The sublimation rate by [8] and [9], with $S_o = 1.0$ cal/cm^2 min, is 18 percent larger than when radiation is neglected for a 100 μ sphere, and 84 percent larger for a 1 mm diameter sphere.

6. Atmospheric Pressure

Rates of sublimation at different elevations may be compared by evaluating [5] and [8] for the corresponding ambient pressures. Only two parameters in equation [5] vary significantly with pressure. Ventilation factors change due to the variation of the kinematic viscosity, and diffusivity of water vapor increases with decreasing pressure.

Variations in kinematic viscosity were estimated from the defining relation $\nu \equiv \mu/\rho_a$ where μ, the absolute viscosity of air, is almost independent of pressure. The density of the air was taken as $\rho_a = p/C_v RT_v$ where p is the pressure, C_v is the coefficient of compressibility, R is the gas constant and T_v is the virtual temperature (List 1968). The results (included in the appendix) show that Reynolds number, and thus the ventilation factor [6], decrease with decreasing pressure. The variation in diffusivity with pressure is $D/D_o = (T/T_o)^n(p_o/p)$,

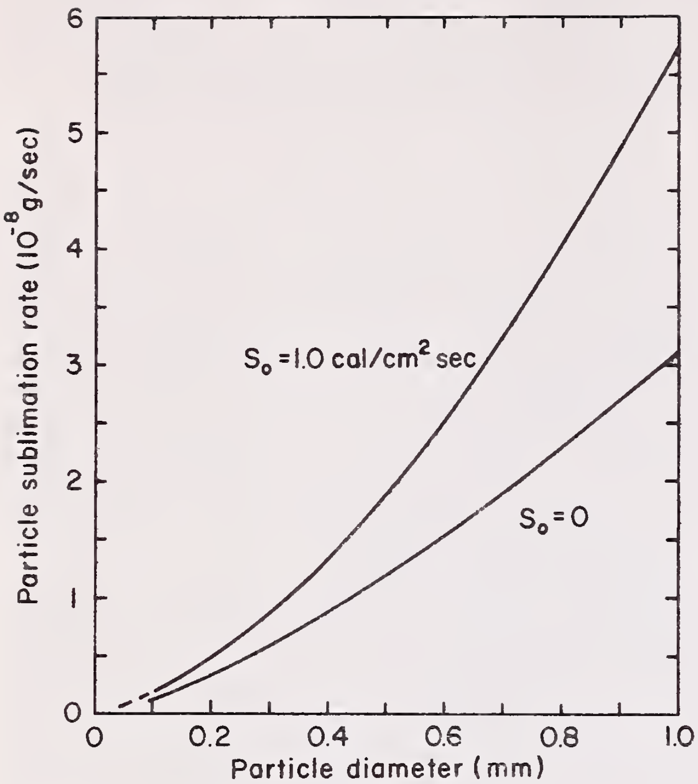


Figure 4.--Solar radiation is more effective in increasing the sublimation rates of larger particles, according to this evaluation of equations [8] and [9] for -20° C. at 1000 mb. The vapor density was 90 percent of the saturation value over ice, and a ventilation rate of 100 cm/sec was assumed for all sizes. The particle surface albedo was taken as $\zeta = 0.5$ and the horizontal surface albedo as $\lambda = 0.8$ in equation [9].

where D_o is 0.180 cm^2 sec^{-1} at p_o = 1000 mb (List 1968). Computations in the appendix are for a temperature of -20° C.

The change in the computed sublimation rate over the elevation range from 0 to 4 km (0 to 13,000 feet) is compared for two particle sizes in figure 5. Substantial increases in sublimation rate are predicted for higher elevations, and the increase is more significant for smaller particles. The increase in diffusivity outweighs the reduction in ventilation rate due to decreased density of air at higher elevations.

Besides changes in ventilation factor and diffusivity, comparisons employing equation [8] must account for the increase in direct and diffuse solar radiation at higher elevations. The mean radiation flux increases by two to three times between sea level and an elevation of 4 km (Geiger 1965). Figure 6 compares sublimation rates at sea level (1000 mb) with those at 2.5 km (750 mb) and 4 km (600 mb). Estimates of S_o, the radiation flux, are listed with the corresponding atmospheric pressure. Of course, holding temperature and humidity constant over this elevation range is not realistic, but figure 6 does show that the net effect of decreasing atmospheric pressure is an increase in sublimation rate at higher elevations, if ambient temperature and humidity are the same.

7. Turbulence

Atmospheric turbulence levels, under conditions of interest here, are about 10 percent for streamwise velocity fluctuations, using approximations suggested by Lumley and Panofsky (1964). (The turbulence level is the ratio of root-mean-square fluctuating velocity to free stream velocity.) Measurements summarized by [6] are for turbulence levels of 1 to 3 percent, as noted before. The question is whether the higher natural turbulence level would cause a greater particle sublimation rate by providing more ventilation than predicted by [6].

If a snow particle followed all turbulent accelerations of air motion exactly during wind transport, then the relative velocity would just equal the particle fall velocity due to gravitational acceleration. However, the particle to air density ratio is close to 1000, assuming the windblown snow particle has a density

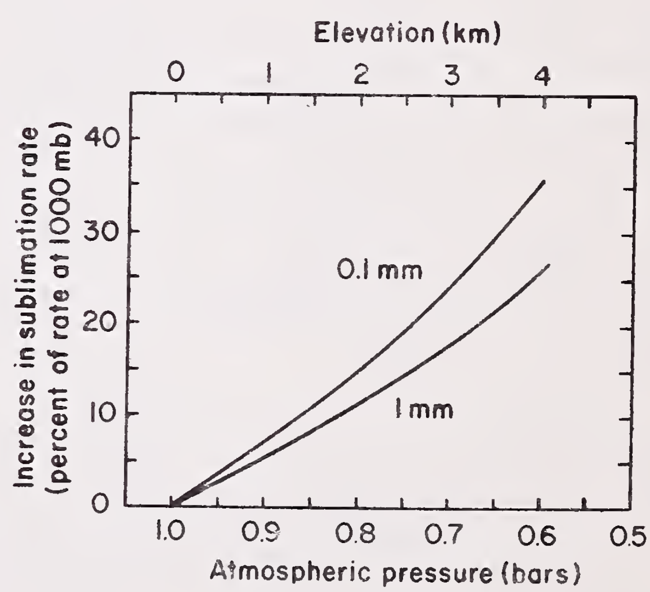


Figure 5.--The sublimation rate for an ice sphere is more rapid at higher elevations according to equation [5] evaluated for -20° C. ambient temperature and a water vapor density equal to 90 percent of the saturation value over ice. A 100 cm/sec ventilation rate was assumed for both particle diameters.

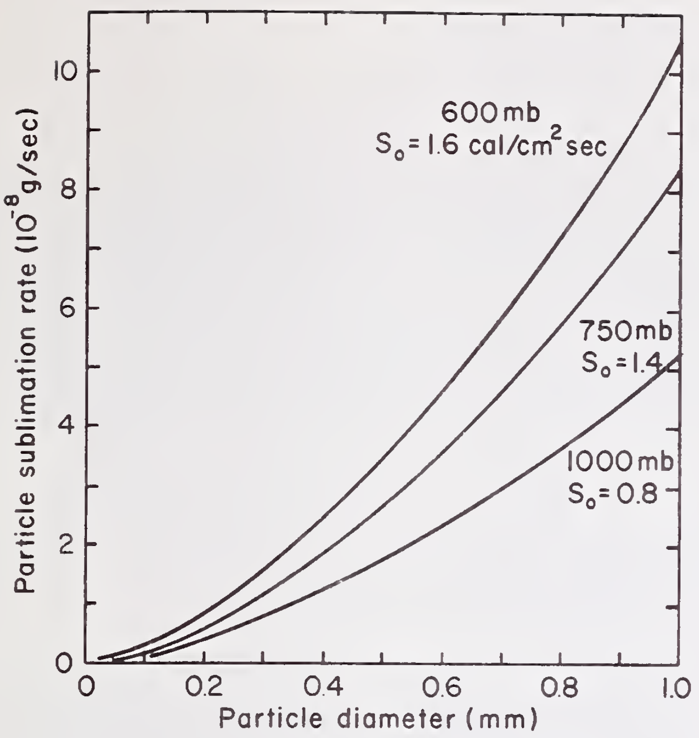


Figure 6.--When the increase of solar radiation at higher elevations is included, equations [8] and [9] provide this picture of particle sublimation rates. Ambient conditions again were -20° C. and 90 percent of water vapor density at saturation over ice, with a ventilation rate of 100 cm/sec for all particles.

near 0.9 g/cm^3 (ice). Accelerations required to overcome particle inertia, and cause it to follow the air velocity fluctuations, must be created by additional drag forces. These can only occur if the relative velocity between the particle and air increases. Thus, the instantaneous relative velocity, u_R (defined by $u_R \equiv u_p - u$, the difference between particle and air velocities) is a fluctuating vector in turbulent flow. It becomes equal to the particle fall velocity ($u_p = w$) when the air is calm ($u = o$).

Friedlander (1957) showed that the mean square relative velocity characterized convection effects on heat and mass transfer from particles suspended in turbulent flow. By making some rather restrictive assumptions, Tchen (1947) derived equations of motion from a balance of forces on the suspended particle. Hinze (1959) and Soo (1967) have both summarized his work. A generalized solution to these equations of motion (Chao 1964) gives the mean square relative velocity in terms of the mean square fluid velocity, the turbulent energy spectrum, and a quantity determined from the stochastic equation of relative motion. This relation predicts that particles of a given size will have an energy spectrum that coincides with the energy spectrum of fluid motion up to some frequency, above which "slip," or relative motion, increases with increasing frequency of turbulent fluctuations.

With the energy spectrum suggested by Lumley and Panofsky (1964) for horizontal gustiness in the atmosphere, Chao's procedure indicates that a 100 μ snow particle would completely follow all turbulent fluctuations. A 1 mm particle would exhibit "slip," that is, it would not follow wind fluctuations, for frequencies higher than 1/10 cycle per second. Csanady (1963) derived a similar equation for mean square relative velocity, by which he calculated that a 0.5 mm dust particle would follow all atmospheric eddies with a vertical dimension greater than 10 m.

The above calculations are based on some rather tentative assumptions, including the absence of particle interactions and particle spin created by the shear flow. More restrictive, drag forces are assumed to obey Stokes' Law, which, strictly speaking, would limit application to situations where the particle Reynolds number is unity or less. An opportunity and need for well designed experiments under atmospheric conditions surely exists here.

Turbulence tends to increase sublimation by increasing the relative velocity above the particle fall velocity. However, the calculations described above do not indicate that this increase is large for the normal size range of windblown snow particles.

B. A Nonspherical Particle

The fine, delicate structures called snowflakes, created during precipitation, are certainly not well approximated by a sphere. As noted in the introduction, these beautiful forms are quickly destroyed by mechanical forces, when wind begins to move deposited snow. This action reduces both the fine structure and the maximum dimensions of the particles, so that the spherical approximation becomes somewhat more realistic. The photomicrographs presented by Budd et al. (1966) and Kojima (1969) all show windblown snow particles without any of the fine dendritic snowflake structure. However, the particles have irregular surfaces and shapes and are more often oblong that spherical.

By the analogy between water vapor diffusion and electrostatics, it is relatively easy to estimate how sublimation rate is affected by deviations from the spherical toward more oblong particle shapes. The calculations in this section show that these deviations tend to increase sublimation rates, but the effect is not large.

1. Diffusion Shape Factor

According to McDonald (1963), the first treatment of the analogy between diffusion and electrostatic fields should be attributed to Maxwell, while Houghton was first to use the analogy in investigating ice-crystal growth. Equation [1], which was derived from this analogy, can be modified to account for particle shape by replacing the radius r with C, the electrostatic capacitance of a conductor with identical size and shape. Then [1] becomes $dm/dt = 4\pi CD(\rho - \rho_r)$. McDonald has verified this analogy for several shapes which also have a theoretical solution in terms of electrostatics, and he measured the value of C for a number of crystal shapes which do not have theoretical solutions.

The four cases in which C may be determined theoretically are (McDonald 1963):

(a) sphere, radius r, $C = r$
(b) thin circular disk, radius r, $C = 2r/\pi$
(c) prolate spheroid, major and minor semi-axis b and c, $C = A/\ln[(b + A)c]$ where $A = (b^2 - c^2)^{1/2}$.
(d) oblate spheroid of major and minor semi-axis b and c. $C = Ae/\sin^{-1}e$ where $e = (1 - c^2/b^2)^{1/2}$.

The limiting value for a thin prolate spheroid, $C = b/\ln(2\,b/c)$ was found to adequately represent the needle crystal form where b is half the needle length and c is the radius of the midsection. McDonald's conclusion was that errors from approximating the true value of C by theoretical values are less than many of the other uncertainties in predicting crystal growth or sublimation.

The sphere has the smallest surface area per unit mass. Any deviation from the spherical shape will increase the area-to-mass ratio, and will result in a larger sublimation rate for a given particle mass. In figure 7, values of C/r are plotted as a function of the ratio of major to minor semi-axis, ξ for a prolate spheroid (formed by revolution of an ellipse about its major axis). For a particle with the same mass as a sphere of radius r, the rate of mass transfer by diffusion would increase 25 percent when the ratio of major to minor semi-axis is 5.

2. Sublimation Equation

When we consider the additional factor of particle shape, equation [8] may be modified to provide a more realistic model. Similar to [3], we have

$$dm/dt = 4\pi DCF_m\,(\rho - \rho_r) \qquad [10]$$

and in place of [7],

$$L_s \frac{dm}{dt} = 4\pi KCF_h\,(T_r - T) + Q \qquad [11]$$

where F_m and F_h are the ventilation factors comparable to 1/2 (Sh) and 1/2 (Nu), respectively, and the radius has been replaced by C, the shape factor. Again as outlined by Thorpe and Mason (1966), the Clausius-Clapeyron equation is used to approximate the vapor density difference, and temperature differences are assumed small. The result analogous to [8] is

$$\frac{dm}{dt} = \frac{4\pi C(\rho/\rho_{sT} - 1) - (Q/KT)\left(\frac{L_s M}{RT} - 1\right)\frac{1}{F_h}}{\frac{L_s}{KT}\left(\frac{1}{F_h}\right)\left(\frac{L_s M}{RT} - 1\right) + \frac{1}{D\rho_{sT}\,F_m}} \qquad [12]$$

Ventilation factors, F_h and F_m, must be evaluated by experiment for a particular particle shape C. From the work of Thorpe and Mason, these factors are expected to have the forms

$$\left.\begin{aligned} F_h &= A + B\,(Pr)^{1/3}(Re)^{1/2} \\ F_m &= A + B\,(Sc)^{1/3}(Re)^{1/2} \end{aligned}\right\} \qquad [13]$$

where A and B are constants.

The radiation term, Q cannot be used in the form given in [9], but must take into account the projected area corresponding to the particle shape. Thus Q takes the form

$$Q = (1 - \zeta)\,A_1 S_o + (1 - \zeta)\lambda A_2 S_o \qquad [14]$$

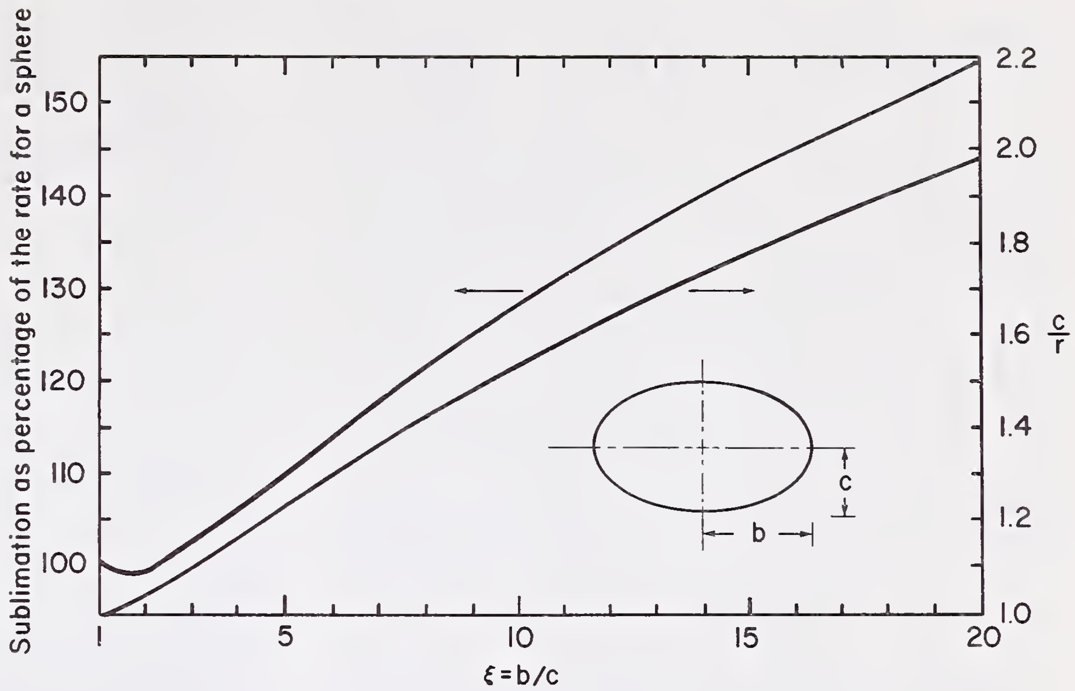
where A_1 and A_2 are the projected areas of the particle perpendicular to the direct and reflected solar radiation, respectively. Values for A_1 and A_2 depend on the average particle orientation with respect to the horizontal, and on the sun angle.

3. Comparison with a Spherical Particle

Sublimation rates predicted by [8] for the spherical particle and by [12] for the nonspherical particle cannot be compared exactly, because values for the wind factors F_h and F_m are, in general, not known for nonspherical particles. However, at least some idea of the uncertainty can be gained by considering the extreme case of an infinite circular cylinder. Work in hot-wire anemometry (Hinze 1959) has established that, for a circular cylinder with major axis at right angles to the flow,

$$(Nu) = 0.42(Pr)^{1/5} + 0.57(Pr)^{1/3}(Re)^{1/2}$$

Figure 7.--The sublimation rate of a prolate spheroid of ice, computed from equation [12], is plotted here as a percentage of the rate for a sphere of equal mass. The variation of C/r (the mass diffusion shape factor) is also shown.



At temperatures and pressures examined here, the values of (Pr) are near unity, so that the relation for an infinite cylinder is approximately

$$(Nu) \simeq 0.42 + 0.57(Re)^{1/2}$$

compared to the value from [6] for the spherical case ($\xi = 1$)

$$(Nu) = 1.88 + 0.58(Re)^{1/2}$$

Assuming the oblate spheroid approximates an infinite circular cylinder when the ratio of major to minor axis is $\xi = 10^3$, the value of C/r is 13.2 from the limiting approximation $C = b/\ln(2b/c)$. Then if the value of (Nu) varies as C/r, for a ratio of $\xi = 20$, the Nusselt number would be $(Nu) \simeq 1.76 + 0.58(Re)^{1/2}$, only slightly less than the spherical case. The value of Reynolds number (Re) is taken as $2cV/\nu$, defined by the minor axis of the spheroid (for the long axis perpendicular to the flow). From these approximations, it appears that particle shape has very little effect on the ventilation factor.

Again taking the standard conditions for these calculations (T = -20° C., p = 1000 mb, (RH) = 90 percent) with a steady mean ventilation rate of 100 cm/sec, evaluation of [12] with Q = 0 provides the comparison in figure 7. For a ratio of major to minor axis of 5, the sublimation rate without radiation is only about 10 percent larger for the spheroid than for a sphere of the same mass. When radiation is added, $(Q \neq 0)$, the effect of particle shape on sublimation rate depends on the particle's orientation with respect to the sun. The maximum projected area of a prolate spheroid increases as the cube root of eccentricity, so a spheroid with $\xi = 5$, oriented with its long axis perpendicular to the sun's rays, intercepts 1.71 times the radiation intercepted by a sphere of the same volume. A spheroid with the volume of a 100 μ sphere, and with $\xi = 5$, would have almost twice the sublimation rate of the sphere, when oriented so that the shape effects were maximum. However, if the particle orientation during turbulent suspension is random, which seems most likely, the shape effect on radiation interception should disappear.

C. Summary

The objective of Part I was to explore the relative importance of numerous factors that determine the sublimation rate of a single particle. Some conclusions drawn from the various references and calculations are:

1. Ice bulb temperature is a reasonable assumption for the temperature of a sublimating snow particle, so differences in particle and air temperatures will be small (less than 0.5° C.) under natural conditions of blowing snow.
2. The sublimation rate nearly doubles for each 10° C. rise in ambient temperature in the range between -20° C. and 0° C.
3. Particle sublimation more than doubles when the diameter is doubled. The percentage of particle mass lost, however, is greater for smaller sizes.
4. Sublimation of a single particle is directly proportional to undersaturation of the environment with respect to water vapor, when radiation is neglected.

5. Heat transfer by solar radiation can double the particle sublimation rate computed for transfer by conduction and convection alone.
6. The increase in estimated sublimation rate at higher elevations is large enough to be important. For the same temperature, humidity, and ventilation velocity, the rate at 4 km (13,000 feet) is about 30 percent more than that computed for sea level, neglecting radiation. When solar radiation and its increase with elevation are considered, the rate computed for 4 km is approximately 50 percent higher than at sea level.
7. Atmospheric turbulence should cause sublimation rates higher than those predicted from laboratory studies, but tentative calculations indicate this effect is not large, for snow particles less than 1 mm in diameter. Field measurements are certainly needed to test this conclusion.
8. Particle shape appears to have less effect on sublimation than most of the factors considered. The calculated rate for a spheroid with a ratio of major to minor axis as large as 5, is only 10 percent greater than for a sphere.

If a spherical ice particle, ventilated at 100 cm/sec, starts sublimating in an environment with 90 percent relative humidity at 1000 mb pressure (sea level) and -20° C., without any radiation transfer, then increases in radiation, elevation, turbulence, and eccentricity all tend to increase the sublimation rate. Even if all these factors are neglected, an ice sphere 100 μ in diameter, under the conditions above, has a computed loss of mass equal to approximately 19 percent of its total weight in 1 minute. This calculation indicates that the sublimation of blowing snow is important enough to explore further.

PART II
SUBLIMATION FROM A VOLUME OF AIR AND SNOW

Having considered the sublimation of a single particle, and the importance of factors that govern this rate, the next question is, how do the results of Part I apply to sublimation rates from many snow particles in a volume of air? The two major problems discussed in this part of the argument are (1) effects of particle concentration on transfer of heat, mass, and momentum within the volume; and (2), sublimation rates for a distribution of particle sizes. The results show that even maximum natural concentrations are low enough that particle interactions can be ignored, and particle sublimation estimates derived in Part I apply directly to each particle within the volume of air and snow.

A. Concentration Effects on Particle Sublimation

The expressions in Part I, for sublimation of a single particle, were based on assumptions that humidity and temperature gradients extended to infinity. Additional particles in a unit volume present sources of water vapor closer to the particle in question, and may thus reduce the rate of mass transfer. As concentration increases, the drag force experienced by the particles increases, once the interparticle spacing becomes small enough that the fluid boundary layers around the spheres interact. Surrounding particles represent heat sinks that reduce the temperature gradients around a given particle, and radiation scattered by other particles also modifies the heat transferred to the particle in question. The importance of these effects on sublimation rate depends on the concentration of particles.

1. Maximum Natural Concentrations

Estimates of the concentration of snow in air during transport are available from the experiments of Budd et al. (1966) in Antarctica. If the lowest centimeter of snow drift is neglected, concentrations approach the mass density of air, and since this argument is limited to evaporation of particles in suspended transport, the maximum concentration of snow will be taken as $M \approx \rho_a$. (Below 1 cm, transport is mainly by saltation and creep.) The volume fraction of particles is the ratio of volume occupied by solids to the total unit volume, denoted by ϕ. This ratio, multiplied by particle density (ρ_p), gives the mass con-

centration of snow, $M = \rho_p \phi$. For maximum conditions, where the mass of snow is equal to the mass of air in a unit volume, $\phi = \rho_a/(\rho_a+\rho_p)$. With $\rho_a = 1 \cdot 10^{-3}$ g/cm^3 and $\rho_p = 0.9$ g/cm^3, $\phi \approx \rho_a/\rho_p$, so only about one-thousandth of a unit volume is occupied by snow at maximum concentrations above the one centimeter level.

2. Correlations of Concentration and Transfer Processes

Studies which correlate mass transfer with voidage $(1 - \phi)$ show that even the maximum concentration of snow expected above 1 centimeter would cause no important reduction in mass transfer due to particle interaction. These measurements were made by Chu et al. (1953), Mullin and Treleaven (1962), and are summarized by Soo (1967). Marshal and Langleben (1954) expressed the effect of a cloud of water drops on the growth of an ice crystal by a term equal to the sum of all drop radii in a surrounding unit volume. This approach also leads to the conclusion that the concentrations present during snow transport by wind do not significantly reduce mass transfer from that of an isolated sphere.

The correlation of drag coefficient with Reynolds number, at the voidage level corresponding to maximum snow concentration, indicates no measurable increase in drag at Reynolds numbers less than 200 (Soo 1967, fig. 5.2). If drag, and thus relative velocity, are not altered, then convection effects on particle sublimation are independent of snow concentration under natural conditions.

No studies that correlated voidage with conduction and convention of heat were found. However, no important changes in these modes of heat transfer are expected, even at maximum concentrations of snow in turbulent suspension. This inference depends on the close analogy between mechanisms of heat and mass transfer during particle sublimation. The nearly identical values of Nusselt and Sherwood numbers, noted in [6], strongly support the conclusion.

Heat transfer by radiation is a completely different process, and the argument reaches no clear conclusion in this case. If each snow particle in a volume of air receives only radiation from outside the volume, and none of the radiation scattered by surrounding particles, the process is called "single scattering," and heat transfer by radiation is independent of particle concentration. Multiple scattering occurs within the volume when the particles interact. Based on remarks by van de Hulst (1957), single scattering probably applies over most of the natural snow concentration range, and can certainly be made to apply in calculations, by limiting the dimensions of the volume considered.

In summary, concentrations of natural windblown snow particles are small enough that particle interactions can be ignored for estimates of the rate at which snow sublimates in a volume of air. Thus, within the limits of optical path length to which single scattering applies, the sublimation rate for a volume is simply the sum of individual particle rates. If the mass of snow per unit volume (M) is composed of N uniform spheres, each with mass, m, sublimating at a rate, dm/dt, then a good approximation of the total sublimation from particles in the unit volume would be $N dm/dt = (M/m) dm/dt$.

B. Effect of Nonuniform Particle Sizes

Size distributions of windblown snow particles measured by Kojima (1969), Budd et al. (1966), and Lister (1960), show that sizes range from a few microns up to 300 μ or more. The more recent studies also show skewness of the size distributions toward smaller particles. Because the sublimation rate depends strongly on particle size, it is important to take the distribution of sizes into account in this model.

1. General Relationships for Nonuniform Sizes

Suppose a unit volume of air-snow mixture contains N snow particles which have a total mass M. A nominal particle diameter, x, may be defined so that the particle mass is given by $m = \rho_p(\pi x^3/6)$. Let some function $f(x)$ describe the frequency distribution of nominal diameters in the unit volume. Then the number of particles with nominal diameters in the range x to (x + dx) is $N f(x)dx$, the total number of particles times the relative frequency for that increment of the particle size distribution. Multiplying the mass of a particle with diameter x and the number of particles in this size increment gives the mass of all particles with diameters from x to (x + dx)

$$M_x = (\rho_p \pi x^3/6) N \; f(x)dx \qquad [15]$$

Integrating [15] with respect to particle size, and rearranging terms, the total number of snow particles in the unit volume is

$$N = M/[(\rho_p \pi/6) \int_0^\infty f(x)x^3 dx] \qquad [16]$$

This relation determines the number of particles in a unit volume of air and snow when the mass concentration and particle size distribution are known.

Let $(dm/dt)_x$ denote the sublimation rate for a particle with nominal diameter in the range from x to (x + dx). Then the rate of sublimation, $(dM/dt)_x$ for the mass associated with this particle size increment, is equal to the number of particles of this size, multiplied by the single particle sublimation rate,

$$(dM/dt)_x = (dm/dt)_x N f(x) dx \qquad [17]$$

Summing rates from all particle size increments gives the sublimation rate for the unit volume, so that by integrating [17] with respect to particle diameter,

$$(dM/dt) = N \int_0^\infty (dm/dt)_x\, f(x) dx \qquad [18]$$

This is a general expression for the rate at which a mass of nonuniform particles sublimates in a unit volume of air and snow. The functional form of $(dm/dt)_x$ was discussed in Part I. Possible forms for the distribution function, $f(x)$ are considered below.

2. Specific Size Distributions

Studies that report size distributions for drifting snow are too few to assure a general distribution function. Several reasonable forms are discussed here, and evaluation of [18] with the two-parameter gamma function suggested by Budd (1966) provides an example.

An exponential function that depends on precipitation rate and crystal type describes the size distributions of aggregate snowflakes during precipitation, according to measurements by Gunn and Marshall (1958) and Ohtake (1970). Within a few hours after snowfall, the number of windblown snow particles in a size class decreases exponentially with increasing size.[4] This work shows that, after several hours of snow drifting without precipitation, a logarithmic-normal function provides a better description of the drift particle size distributions.

Budd (1966) shows that the log-normal function gives a fair approximation to the size distributions reported in Budd et al. (1966).

[4] *Schmidt, R. A. Distributions of snow particle size during wind transport over a ridge. (Manuscript in preparation at Rocky Mt. Forest and Range Exp. Stn., Fort Collins, Colo.)*

For his calculations of drift content, however, Budd used a two-parameter gamma function which gave as good a fit to the size distributions, and was mathematically more convenient for his purpose. Following his suggestion, the distribution of particle size is specified as

$$f(x)dx = \frac{1}{\beta^\alpha \Gamma(\alpha)} e^{-x/\beta} x^{\alpha-1} dx \qquad [19]$$

where α and β are the parameters of the distribution, such that $\alpha\beta$ is equal to the mean, and $\Gamma(\alpha)$ is the complete gamma function. The shape parameter is α, and β is the scale parameter (Thom 1958).

With a specified particle size distribution, [16] will yield an expression for the number of nonuniform particles in a unit volume. Substituting the expression [19] for $f(x)$ in [16] gives the number of particles in terms of mass concentration and particle size:

$$N = M/\left[(\rho_p \pi/6)\int_0^\infty \frac{1}{\beta^\alpha \Gamma(\alpha)} e^{-x/\beta} x^{(\alpha+3)-1} dx\right] \qquad [20]$$

By the definition of a gamma variate and the fact that the area under a distribution function must equal unity, [20] can be solved to give the number of particles per unit volume in terms of the mass concentration of snow and the parameters of the size distribution,

$$N = M(6/\rho_p \pi)[\Gamma(\alpha)/\Gamma(\alpha + 3)]\beta^{-3} \qquad [21]$$

The sublimation rate of this particular distribution of particles is determined from [18]. The integration cannot be performed, however, until a relationship between particle size and ventilation rate is formulated.

3. Particle Size and Ventilation Velocity

A ventilation rate equal to the particle fall velocity was assumed in Part I, when the effects of turbulent air accelerations were neglected. Continuing with this tentative assumption, a deterministic relationship between particle size and fall velocity will admit integration of [18].

Mellor (1965) summarizes the problem of expressing the fall velocities of drifting snow in terms of particle size. For diameters less than 60 μ, particles obey Stokes' Law reasonably well, and fall velocity is proportional to the second power of diameter. In the range between 100 μ and 1 mm, fall velocity is directly proportional to the first power of diameter. Although drifting snow particles larger than

1.5 mm usually are not transported by turbulent suspension, such particles fall at speeds proportional to the square root of diameter. Recognizing that transitions must occur between these ranges, Mellor suggested the relation, $w = w_o \exp(-x_o/x)$, for fall velocities of drift particles less than about 5 mm diameter. In this expression, w_o is the terminal velocity of large particles with x_o diameter.

In his analysis, Budd (1966) assumed the fall velocity of all suspended drift particles was $w = C_2 x$, noting that this relationship would be in error for the small particles. Depending on particle shape, the proportionality constant C_2 was between 2440/sec and 3880/sec, the latter applying to spherical particles. This assumption is used in the following calculation, which does not warrant the more complicated relation suggested by Mellor.

4. Calculation of Volume Sublimation Rate

To estimate a sublimation rate by [18], first the total mass of snow in the volume of air is required. With this value for M, and a size distribution function, such as [19], the number of particles in the volume can be calculated from [16]. Finally, an expression for single-particle sublimation, such as [12] derived in Part I, is combined with some relation between particle size and ventilation velocity to give an integrable expression for $(dm/dt)_x$ in [18].

As an example, suppose the volume of interest is a horizontal layer of snow and air, 1 cm thick and 1 m square. (The reason for considering these dimensions becomes apparent in Part III.) If this layer is located 1 m above a horizontal snow surface, then a snow concentration of 1 g/m^3 might be expected for 15 m/sec wind velocities, according to Antarctic measurements by Budd et al. (1966). The mass of snow (M) in the given volume would be 0.01 g.

If the distribution of sizes is approximated by [19], then parameters α and β must be specified. Budd (1966) obtained values of the shape parameter (α) near 15. Calculations here are for several mean diameters($\bar{x} = \alpha\beta$), with $\alpha = 15$, so that only the scale parameter (β) varies. These distributions are plotted in figure 8.

Nighttime, or minimum radiation transfer ($Q = 0$) is assumed, and particle shape effects are neglected so that equation [5] is used in place of [12] to express single particle sublimation. If [5] and [6] are combined, the expression is

$$\frac{dm}{dt} = C_1[1.88x + 0.58x\,(Vx/\nu)^{1/2}] \qquad [22]$$

where the mathematical presentation is considerably reduced by the substitution

$$C_1 \equiv \frac{\pi\sigma}{\frac{L_s}{KT}\left[\frac{L_s M}{RT} - 1\right] + \frac{1}{D\rho_s(T)}} \qquad [23]$$

Assuming that ventilation velocity is equal to particle fall velocity ($V = w$), and further, that fall velocity is directly proportional to particle diameter ($w = C_2 x$), the rate of sublimation for a spherical particle is

$$(dm/dt)_x = C_1[1.88x + 0.58\,(C_2/\nu)^{1/2}x^2] \qquad [24]$$

For all particles in the unit volume, the sublimation rate is estimated by

$$(dM/dt) = [NC_1/\beta^\alpha\Gamma(\alpha)]\int_0^\infty [1.88\,x + 0.58\,(C_2/\nu)^{1/2}\,x^2]e^{-x/\beta}\,x^{\alpha-1}dx \qquad [25]$$

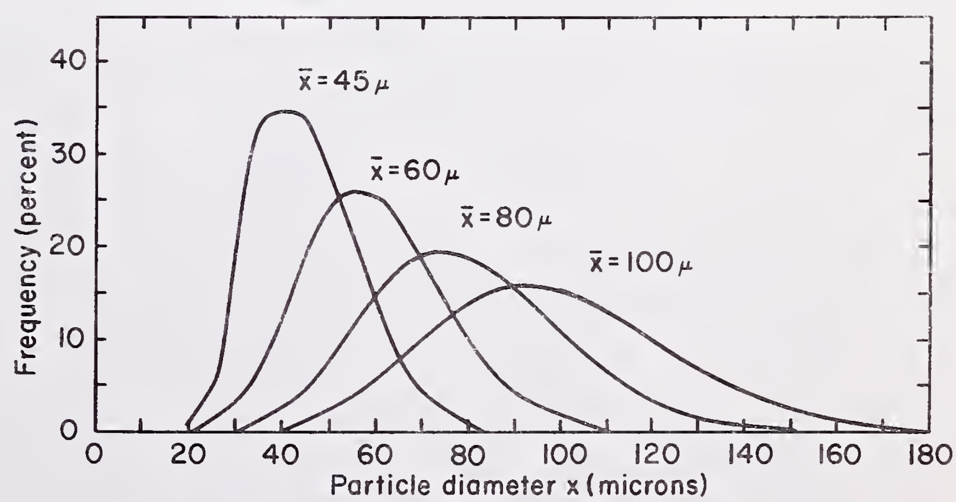


Figure 8.--These particle size distributions are used for the calculations in table 1 and figure 9. The two-parameter gamma function has a mean $\bar{X} = \alpha\beta$. The value $\alpha = 15$ was taken for all distributions.

where N is computed from [21] and C_1, defined by [23], is constant for a given set of temperature, pressure, and humidity conditions.

The results of evaluating [25] for the several size distributions, sublimating at -20° C., 1000 mb and 90 percent relative humidity, are shown in figure 9. A value of C_2 = 3880/sec was used for spheres, and particle density was assumed equal to 0.92 g/cm^3. Sublimation rate increases markedly when the same mass of snow is composed of particles with smaller and smaller mean diameters. Skewness of size distributions toward smaller diameters has the same effect of increasing surface area and thus sublimation.

An average particle sublimation rate may be computed by dividing the total number of particles into the volume sublimation rate. Equating this value and the righthand side of [24] gives the diameter of a particle with the average sublimation rate. These calculations were carried out for the four distributions in figure 8 (table 1). Average sublimation diameters are nearly the same as the distribution mean diameters for these examples.

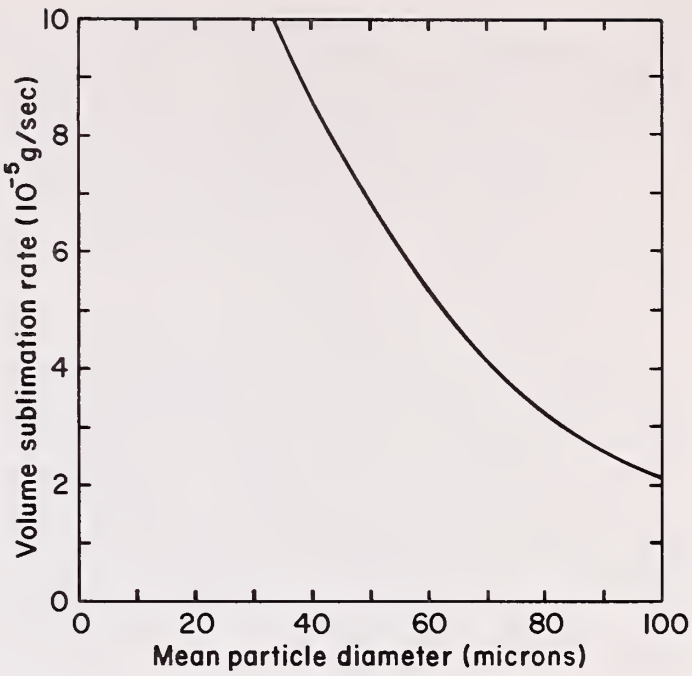


Figure 9.--The sublimation of a given mass of snow greatly increases as the mean particle diameter decreases according to this plot. Equation [25] was evaluated with the size distributions in figure 8 to estimate the sublimation rate of 0.01 gram of snow in a 10^4 cubic centimeter volume at -20° C. and 1000 mb. Water vapor density was again 90 percent of saturation over ice.

C. Summary

Concerning the effect of snow concentration on transfer processes, the following conclusion was drawn:

1. For heights greater than 1 cm above the snow surface, natural windblown snow concentrations are too low to have an effect on heat, mass, and momentum transfer, so that the relations developed for a single sphere in Part I apply to each particle in a volume of drifting snow. It follows that the total sublimation of snow in a volume is the sum of the sublimation rates for each particle as if it were isolated.

Introduction of a particle size distribution produced calculations summarized by figure 9, which led to the conclusion that:

2. Sublimation of a mass of snow in a volume of air is extremely sensitive to the distribution of particle sizes, due to the surface area relationship. However, there is apparently little error in computing sublimation for N spheres with the same diameter as the mean of the distribution, at least for the distribution function and sizes considered.

Table 1.--Comparison of average sublimation diameters, $\bar{x}_s$

Mean diameter ($\bar{x}$ in μ)	Total number N	Volume sublimation rate (dM/dt) in g/sec	Average particle rate (dM/dt)/N in g/sec	Average sublimation diameter ($\bar{x}_s$ in μ)
100	17,220	$2.19 \cdot 10^{-5}$	$1.272 \cdot 10^{-9}$	102
80	33,690	$3.17 \cdot 10^{-5}$	$9.409 \cdot 10^{-10}$	81
60	79,850	$5.18 \cdot 10^{-5}$	$6.487 \cdot 10^{-10}$	61
40	189,300	$8.59 \cdot 10^{-5}$	$4.538 \cdot 10^{-10}$	45

PART III
SUBLIMATION IN A VERTICAL COLUMN OF DRIFTING SNOW

Natural concentrations of drifting snow decrease very rapidly with height, and average particle size also becomes smaller in the first few meters above a snow surface. When these changes are expressed as a function of the mean wind velocity profile, the volume sublimation rate (Part II) can be integrated with height, to predict the sublimation rate in a vertical column of drifting snow. Evaluation of this general, steady-state equation indicates that water losses through the sublimation process are potentially quite important in a water balance. The vertical fluxes of heat and water vapor required to maintain a steady state provide an interesting set of conditions for the vertical profiles of temperature and humidity.

A. General Equation for Steady-State Sublimation

When snow is drifting, speeds are high enough that the vertical profiles of wind in the first 10 meters above a horizontal surface are very well expressed by

$$\bar{u} = (u_*/k)\ \ln(z/z_o) \qquad [26]$$

according to the Antarctic measurements. The mean horizontal wind component ($\bar{u}$) is proportional to the natural logarithm of height (z) measured vertically from the surface. A friction velocity($u_* \equiv \sqrt{\tau_o/\rho_a}$), defined by the ratio of surface shear stress to air density, and von Karman's constant (k = 0.4) determine the proportionality factor. The integration constant (z_o) is often called a "roughness parameter" because its value depends on conditions of the horizontal surface.

For this windspeed profile relation, the eddy viscosity is $\varepsilon = ku_*z$. Shiotani and Arai (1953), and Loewe (1956), assumed the eddy diffusivity of uniform snow particles was equal to the eddy viscosity. Then, equating the vertical flux of snow by turbulent exchange with the downward flux due to gravity provides an expression for the relative number of particles at any height, in the form

$$N_z/N_Z = (z/Z)^{-w/ku_*} \qquad [27]$$

The number of particles (N_Z) at some reference height (Z) must be known to evaluate N_z from [27]. This equation also holds for the mass concentration ratio as long as all particles have the same mass. (Sommerfeld and Businger (1966) presented measurements which cast some doubt on the assumption that particle diffusivity and eddy viscosity are equal. However, as Radok (1968) points out, their measured particle fall velocities seem unreasonably high, which could account for the discrepancy. If particle "slip" is negligible, the assumption should be valid, and the argument will continue with caution.)

Budd (1966) assumed that particles in each small size class were distributed with height according to [27]. Photoelectric particle counter measurements [5] support this assumption. When N_{zx} denotes the number of particles at some height z, in a nominal diameter class x to (x + dx), this assumption is formalized by

$$N_{zx} = N_{Zx}\ (z/Z)^{-w/ku_*} \qquad [28]$$

Then if $f(x)$ is the distribution function of nominal diameters in a unit volume at the reference height (Z), the number of particles in a size class at that height is $N_{Zx} = N_Z f(x)\,dx$, and thus

$$N_{zx} = N_Z (z/Z)^{-w/ku_*}\ f(x)\ dx \qquad [29]$$

For all particles at height z, with nominal diameters between x and (x + dx), the sublimation rate is $(dM/dt)_{zx} = N_{zx}\ (dm/dt)_x$, or

$$(dM/dt)_{zx} = N_Z (dm/dt)_x (z/Z)^{-w/ku_*} f(x)\ dx \qquad [30]$$

Integrating this expression with respect to nominal diameter (x) gives a relationship for volume sublimation rate at height z as

$$(dM/dt)_z = N_Z \int_o^\infty (dm/dt)_x (z/Z)^{-w/ku_*}\ f(x)dx \qquad [31]$$

The final step toward a general steady-state expression for sublimation within a vertical column of drifting snow is to apply [31] to a small volume with vertical increment dz, and integrate with height, to get $(dM/dt)_c$, the total rate for the column as

$$(dM/dt)_c = N_Z \int_o^z \int_o^\infty (dm/dt)_x\ (z/Z)^{-w/ku_*} f(x)\,dx\,dz \qquad [32]$$

[5] *Schmidt, R. A. Distributions of snow particle size during wind transport over a ridge. (Manuscript in preparation at Rocky Mt. Forest and Range Exp. Stn., Fort Collins, Colo.)*

Evaluation of this expression requires the following:

a. the mass or number concentration at the reference height, to provide N_Z ;
b. a functional expression for the size distribution, $f(x)$;
c. at least two points on the wind profile, to determine the friction velocity, u_*;
d. some expression for particle fall velocity (w) in terms of the nominal particle diameter (x);
e. an integrable relation for the single particle sublimation rate.

B. Calculations for Constant Temperature and Humidity

The objective of evaluating [32] at this point is to demonstrate the potential magnitude of drift snow sublimation, even under relatively cold, moist conditions. Calculations are for a vertical column, 1 meter square in horizontal cross section and extending from 1 cm to 10 m above the horizontal snow surface. Average windspeed at 10 m is 15 m/sec, friction velocity (u_*) is 52 cm/sec, and Z = 1 m is specified for a reference height. As in Part II, the mass of snow in a 1 cm layer at this level is assumed to be 0.01 g, with particle sizes distributed as a two-parameter gamma function where $\alpha = 15$, and $\bar{x} = 100\ \mu$. The number of particles (N_Z) in this layer has been calculated as 17,220.

Since sublimation depends strongly on particle surface area, and therefore particle concentration, the vertical drift concentration profile is an important part of sublimation calculations for a column. A general equation for the mass of snow in a layer at any height z may be derived from [29] as

$$M_z = N_Z\ (\pi\rho_p/6) \int_0^\infty (z/Z)^{-w/ku_*} x^3 f(x)\ dx \qquad [33]$$

When Budd's assumptions for fall velocity ($w = C_2 x$) and size distribution [19] are made, the drift concentration profile is given by

$$M_z = M_Z\ (1 - C_2\beta/ku_* \ln z/Z)^{-(\alpha+3)} \qquad [34]$$

Figure 10 compares the average of profiles measured by Budd et al. (1966, p. 95), for 14.5 m/sec mean windspeed, with the concentration profile computed by [34]. To obtain a reasonable prediction, the value C_2 = 2440/sec had to be used. The value C_2 = 3880/sec (for spheres) predicted too rapid a decrease in concentration with height. (One should not conclude too much about particle shape from the comparison, since the assumption $w = C_2 x$ only roughly approximates the true relationship for much of the size range.)

An estimate of the mass transfer shape factor (C in Part I), which corresponds to the value of C_2 = 2440/sec, can be derived from drag coefficient - Reynolds number correlations for irregular particle shapes. One such compilation of measurements is presented in Report 12 of the Subcommittee on Sedimentation (Inter-Agency Committee on Water Resources U.S.A., 1958). Comparisons of fall velocities predicted by $w = C_2 x$ with those given by the drag curve for different particle shapes indicate that the capacitance C would be only about 4 percent greater than for a sphere (C = x/2), even when C_2 = 2440/sec. The conclusion is: even though particle shape plays an important role in determining the drift concentration profile, no large error in sublimation rate is introduced when single particle rates are computed for the spherical case.

This conclusion leads to the particle sublimation rate equation [8], which, with [6], and the assumption $w = C_2 x$, again results

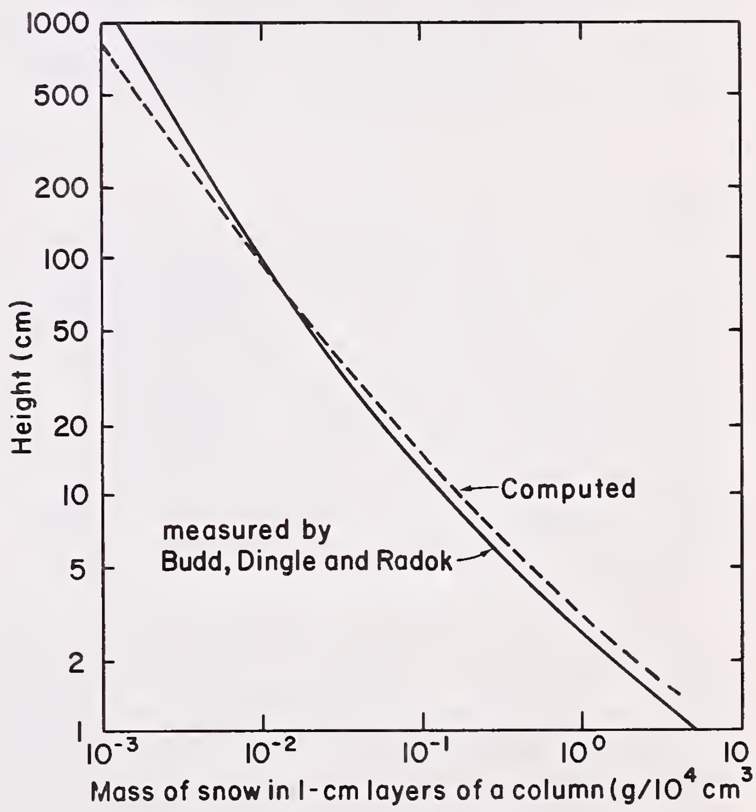


Figure 10.--The snow concentration profile computed from equation [34] with C_2 = 2440/sec compares reasonably with the average drift density profile reported by Budd et al. (1966) for windspeeds near 14.5 m/sec. Values used in the calculations included a 10-m windspeed of 15 m/sec, u_* = 52 cm/sec, α = 15 and $\bar{x}$ = 100 microns. The drift density at the reference level Z = 1 m was assumed to be $M_Z = 0.01\ g/10^4 cm^3$.

in [24] for particle sublimation without radiation ($Q = 0$). When this expression for $(dm/dt)_x$, and the distribution function [19], are substituted in [31] the sublimation within a layer at height z is

$$\left(\frac{dM}{dt}\right)_z = \frac{N_Z C_1}{\beta^\alpha \Gamma(\alpha)} \int_0^\infty [1.88x^\alpha + 0.58(C_2/\nu)^{1/2} x^{\alpha+1}] e^{(C_2 a - 1/\beta)x} dx \quad [35]$$

where the substitution $a = (1\ ku_*) \ln z/Z$ has been used with the identity, $(z/Z)^{-C_2 x/ku_*} \equiv \exp(C_2 ax)$. Evaluating the integral gives

$$\left(\frac{dM}{dt}\right)_z = N_Z C_1 \bar{x}(1 - C_2 a\beta)^{-(\alpha+1)} [1.88 + 0.58(C_2/\nu)^{1/2} (\bar{x}+\beta)/(1-C_2 a\beta)] \quad [36]$$

The final step of integrating [36] with respect to height does not lead to an analytical expression for the total column sublimation. An approximation has therefore been obtained by summing the sublimation rates computed by [36] for each of the layers 1 cm thick and 1 m square that make up the column of drift snow being considered. For this example, the variation of volume sublimation with height is plotted in figure 11, which illustrates the strong influence of the concentration profile. The mass of snow computed from the concentration profile was 14 g for the column, and the estimated sublimation rate $(dM/dt)_c$ was $3.9 \cdot 10^{-2}$ g/sec. Neglecting the strong influence of radiation, even at -20° C. and 90 percent relative humidity, the estimated snow sublimation rate is 0.28 percent each second, which, if conditions were somehow maintained, would be almost 17 percent per minute.

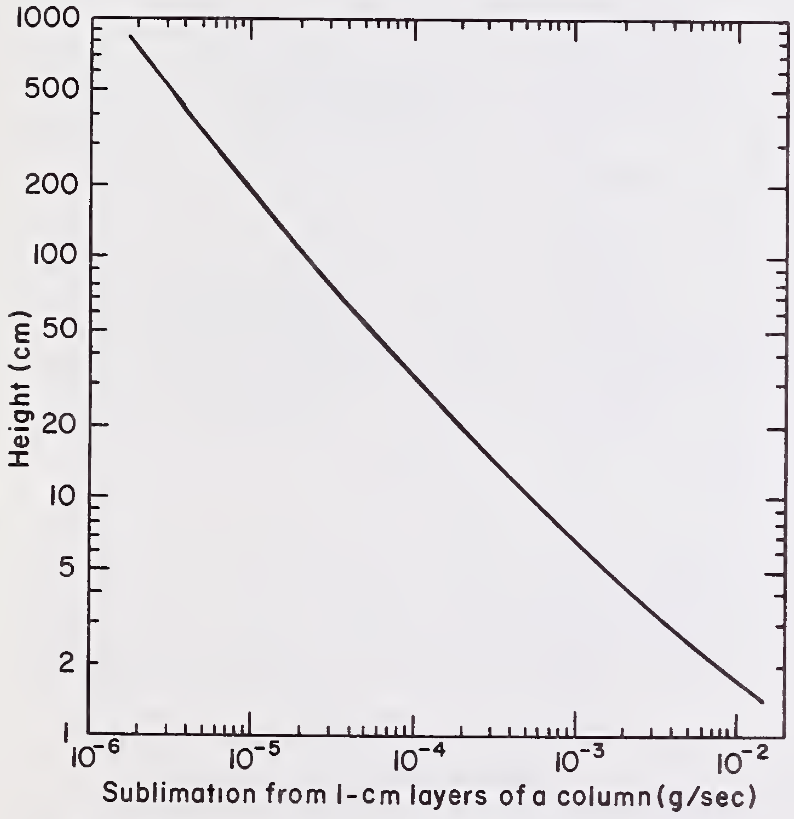


Figure 11.--Assuming the humidity is constant with increasing height (in this case, 90 percent of the saturation value over ice at -20° C.), the sublimation rates in 1-centimeter layers of a column of air and snow show a variation with height similar to the snow concentration profile. Values in this figure were computed by equation [36] for the calculated concentration profile in figure 10.

C. Vertical Fluxes under Steady Conditions

To maintain steady humidity and temperature conditions within the column of drifting snow, water vapor must be removed, and heat added, at rates that balance the sublimation process. With no horizontal gradients of vapor or temperature (steady state), turbulent transfer of vapor to the atmosphere above the column requires a vertical humidity gradient. Heat is added to the column from the snow surface by turbulent exchange along a vertical temperature gradient (forced convection), although both direct and reflected solar radiation may also transfer heat to particles in the column. Thus, radiation and the vertical temperature and humidity profiles interact with the sublimation profile to establish an equilibrium in this steady state model.

1. Humidity Profile

If the humidity of some thin horizontal layer in a column of drifting snow remains constant with time, the sublimation rate of that layer must be matched by an increase in the vertical water vapor flux across the layer. Thus

$$\frac{\partial E}{\partial z} = \left(\frac{dM}{dt}\right)_z \quad [37]$$

where E denotes the vertical vapor flux due to turbulent exchange. Following Priestly (1959), the vertical flux of water vapor, E, is written

$$E = -\rho_a K_w (\partial \bar{q}/\partial z) \quad [38]$$

where q is the specific humidity (the mass of water vapor divided by the mass of moist air) and K_w is the eddy diffusivity for water vapor. Similarly, for momentum flux τ and eddy viscosity (K_m) the defining equation is $\tau = \rho_a K_m(\partial\bar{u}/\partial z)$. Specific humidity may be written as $q = \rho/\rho_a$ and when the density of moist air (ρ_a) is considered constant over the column height, the humidity gradient is $(\partial\rho/\partial z) = \rho_a(\partial q/\partial z)$. If one makes the basic assumption that eddy diffusivity is equal to eddy viscosity $(K_w = K_m)$, the vertical vapor flux, in terms of the humidity and windspeed gradients, is

$$E = \frac{-\tau}{\rho_a}\,\frac{\rho_a(\partial\bar{q}/\partial z)}{(\partial\bar{u}/\partial z)} \qquad [39]$$

Recalling that $u_* \equiv \sqrt{\tau/\rho_a}$, the flux may be rewritten

$$E = -u_*^2(\partial\rho/\partial z)/(\partial\bar{u}/\partial z) \qquad [40]$$

where the profile of specific humidity has been written in terms of vapor density or absolute humidity, ρ. From the logarithmic wind profile, the windspeed gradient is $(\partial\bar{u}/\partial z) = u_*/kz$, neglecting the zero displacement z_o. So, for conditions of interest here (near neutral stability), the vertical vapor flux is estimated from

$$E = -u_* kz(\partial\rho/\partial z) \qquad [41]$$

By definition $\sigma \equiv (\rho/\rho_{sT}-1)$, and thus $\partial\rho/\partial z = \rho_{sT}\,\partial\sigma/\partial z$ so [41] may be written as $E = -(u_* k\rho_{sT})z(\partial\sigma/\partial z)$ and differentiation gives

$$\frac{\partial E}{\partial z} = -(u_* k\rho_{sT})\left(z\frac{\partial^2\sigma}{\partial z^2} + \frac{\partial\sigma}{\partial z}\right) \qquad [42]$$

The equilibrium condition [37] equates [42] with the general expression [31] for $(dM/dt)_z$ to give a differential equation for the humidity profile.

$$\frac{\partial^2\sigma}{\partial z^2} + \frac{1}{z}\frac{\partial\sigma}{\partial z} + \frac{N_z}{u_* k\rho_{sT} z}\int_o^\infty (dm/dt)_x \qquad [43]$$

$$(z/Z)^{-w/ku_*} f(x)dx = 0$$

Single particle sublimation $(dm/dt)_x$ also contains the variable σ, and for the steady state, this equation is an ordinary differential equation of second order, with variable coefficients. If P, Q, and R denote functions of height, and a prime indicates differentiation with respect to z, [43] has the general form

$$\sigma'' + P\sigma' + Q\sigma = R \qquad [44]$$

where $R = 0$ when radiation is neglected.

Perhaps because of the unfortunate combination of assumptions, no solution for the humidity profile was obtained with the assumption $w = C_2 x$ and the two-parameter gamma distribution for $f(x)$. Nevertheless, a general picture of the humidity profile curvature is gained from the vertical vapor flux equation [41].

If humidity decreases in direct proportion to the logarithm of height (say $\rho = \rho_{sT} - A\ln z$), then the humidity gradient is inversely proportional to height $(\partial\rho/\partial z = -A/z)$, and according to [41], the vapor flux is constant with height. However, to exhaust the vapor created by sublimation, E must increase with height, to match $\int^z (dM/dt)_z dz$ as shown by figure 12. Therefore, the humidity profile must be such that $z\partial\rho/\partial z$ increases with height, rapidly at first. Such a profile is sketched in figure 13, along with the logarithmic humidity profile. Since the sources of water vapor are distributed with height, the vertical humidity profile in drifting snow might resemble that in a layer of transpiring vegetation.

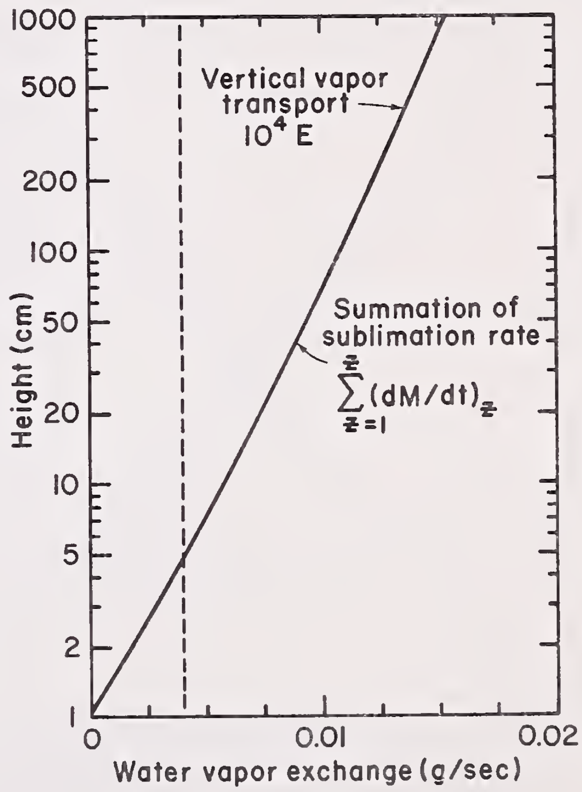


Figure 12.--A logarithmic humidity profile should correspond to constant vertical vapor transport with height. However, the total sublimation increases with height so that such a humidity profile would not indicate the required balance between vertical vapor flux and sublimation rate. The humidity profile used in these calculations was $\sigma_z = \sigma_{100}(1 + 0.217\ln z/Z)$, where σ_{100} is the undersaturation at 100 cm, (taken as 0.1).

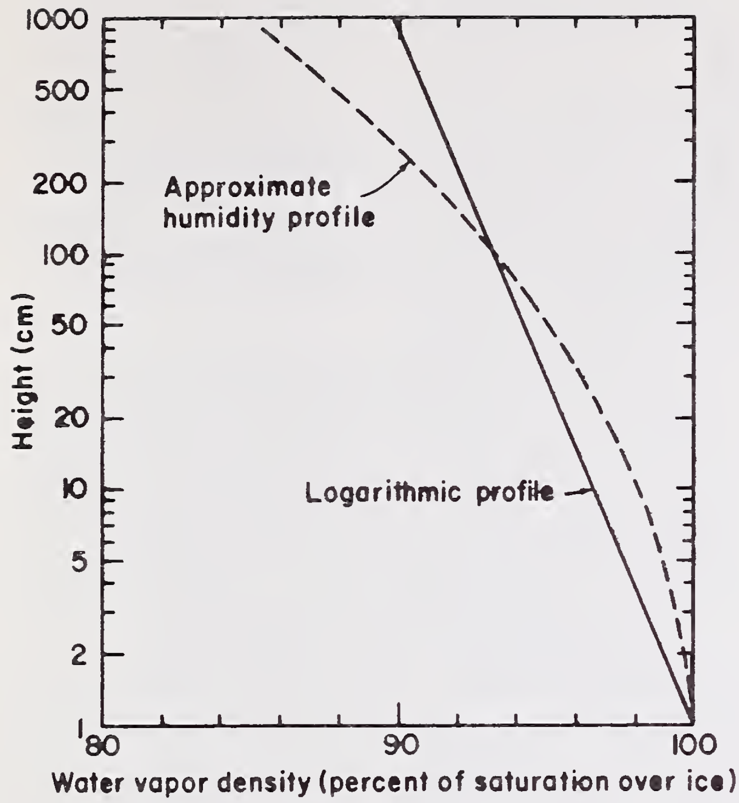


Figure 13.--To maintain a balance between steady-state vertical vapor transport and sublimation of blowing snow, the humidity profile is expected to be similar in shape to the one sketched here.

2. Temperature Profile

For the vertical heat flux (H), a defining equation analogous to [38], is given by Priestley (1959) as

$$H = -\rho_a C_p K_h \left(\frac{\partial T}{\partial z} + \Theta\right) \qquad [45]$$

The turbulent heat-transfer coefficient is K_h, and the adiabatic lapse rate (Θ) is approximately -0.01° C/m. Then, with the logarithmic wind profile and the assumption that $K_h = K_m$, the equation becomes

$$H = -\rho_a C_p u_* kz \left(\frac{\partial T}{\partial z} + \Theta\right) \qquad [46]$$

To maintain a steady sublimation rate within some layer at height z, the amount of heat required is $L_s (dM/dt)_z$. If this heat is transferred to the layer entirely by turbulent exchange, the vertical heat flux is reduced by this amount across the layer so that the equilibrium condition is

$$\frac{\partial H}{\partial z} = -L_s (dM/dt)_z \qquad [47]$$

Differentiating [46] provides the steady-state equation for the temperature profile in a form analogous to [43].

$$\frac{\partial^2 T}{\partial z^2} + \frac{1}{z}\left(\frac{\partial T}{\partial z} + \Theta\right) - \frac{L_s}{\rho_a C_p u_* k}\frac{1}{z}\left(\frac{dM}{dt}\right)_z = 0 \qquad [48]$$

When direct and reflected solar radiation are included, the decrease in turbulent heat flux with height supplies the difference between heat used in sublimation and that transferred to the layer by radiation (Q_z). Then the equilibrium condition is

$$\partial H/\partial z = -[L_s\, dM/dt - Q_z] \qquad [49]$$

where the radiation (Q_z) absorbed by particles in the volume at height z is

$$Q_z = N_Z S_o (1 + \lambda) \int_o^\infty (1 - \zeta)\, \pi/4\, x^2 f(x)dx \qquad [50]$$

if single scattering applies throughout the column. The temperature profile may be obtained from the differential equation,

$$\frac{\partial^2 T}{\partial z^2} + \frac{1}{z}\left(\frac{\partial T}{\partial z} + \Theta\right) - \frac{L_s}{\rho_a C_p u_* kz}\left(\frac{dM}{dt}\right)_z = -\frac{Q_z}{\rho_a C_p u_* kz} \qquad [51]$$

D. Summary

No calculations have been included following the arguments on the humidity and temperature profiles. Several assumptions should be tested experimentally before any large amount of work is expended in such calculations. Therefore, the assumptions and general form of the model are listed here in anticipation of the brief discussion that concludes this paper. Important assumptions for the general steady-state equation are:

a. The sublimation rate of each windblown snow particle is unaffected by particle interactions.
b. Particles of each size class are distributed with height according to the steady-state theory for uniform particles (where eddy viscosity is equal to particle diffusivity).
c. The logarithmic expression [26] describes the vertical wind profile.

The sublimation rate within a unit volume at some height (z) above a horizontal snow surface then has the general form given by [31] as

$$(dM/dt)_z = N_Z \int_o^\infty (dm/dt)_x (z/Z)^{-w/ku_*} f(x)dx$$

A differential equation for the vertical profile of humidity results from the assumptions that:

d. All sublimated vapor is transferred vertically by turbulent exchange.
e. Eddy diffusivity for water vapor is equal to eddy viscosity ($K_w = K_m$).

The humidity profile is obtained by the solution of

$$\frac{d^2\sigma}{dz^2} + \frac{1}{z}\frac{d\sigma}{dz} + \frac{(dM/dt)_z}{u_* \, k \, \rho_{sT} z} = 0$$

Likewise, for temperature, the assumptions are:

f. The heat required for the sublimation process is supplied by radiation, and the vertical transfer by turbulent exchange.

g. The coefficient of turbulent heat exchange is equal to eddy viscosity ($K_h = K_m$).

The differential equation for the vertical profile of temperature is

$$\frac{d^2T}{dz^2} + \frac{1}{z}\left(\frac{dT}{dz} + \Theta\right) - \frac{L_s (dM/dt)_z}{\rho_a \, C_p \, u_* \, kz} = \frac{-Q_z}{\rho_a \, C_p \, u_* \, kz}$$

DISCUSSION

The assumptions used to derive a general relationship for sublimation of drifting snow appear quite reasonable, considering the low natural concentrations and the near-neutral stability likely under these strong winds. (The possible exception, in assuming that eddy viscosity equals snow diffusivity, has been noted.) Many of the suppositions made in order to evaluate the general equation, however, present interesting questions for the experimenter.

First, what is the density of a drifting snow particle? Is it equal to that of ice, as assumed in these calculations? If not, then the surface area of a given mass of drifting snow is larger, and the particle fall velocity is less, than predicted here. Answering this question is fundamental to the study of sublimation from drifting snow.

How is particle fall velocity related to particle size? This question involves the answer to the particle density problem, as well as a correlation between particle shape and fall velocity. In this paper the linear relation, $w = C_2 x$, was used to express the fall velocity in terms of shape and size, but the assumption has an important influence on the computed snow concentration profile. Thus, a more accurate approximation would be worthwhile.

What is the actual ventilation velocity of a suspended snow particle? Is it closely approximated by the particle fall velocity, or should the interaction between particle size and the turbulent energy spectrum be included? Related to these questions is the effect of ventilation on small particles ($Re < 10$). Calculations here have used the same particle sublimation rate throughout the size range, but this may overestimate the sublimation of the small particles.

By the arguments presented in this paper, the vertical profiles of both humidity and temperature are expected to show the influence of sublimation. Simultaneous profile measurements of humidity, temperature, windspeed, and snow concentration under measured conditions of radiation would provide a test of this model. However, several instrumentation problems must be overcome before such measurements are possible.

To assess the hydrologic importance of sublimation from drifting snow within some region, another set of questions requires answers. How frequently do strong winds move snow? What temperature and humidity conditions prevail during these events? What proportion of the drifting occurs at night, and, during the day, how much solar radiation is received?

Probably the greatest shortcoming of this sublimation model is the steady-state assumptions made in all stages of the development. Hopefully, this work will provide some initial level of understanding, but certainly the unsteady nature of the phenomenon must be accounted for, in a true description of sublimation from wind-transported snow.

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APPENDIX

VALUES OF PHYSICAL PROPERTIES OF AIR AND WATER USED FOR CALCULATIONS

Table of Values at 1000 mb

Temperature, T (°C)	K[1]	L_s[1]	D[2]	$\rho_s(T)$[1]	ν	μ[1]	M	R	M/R
	$cal\ cm^{-1}\ sec^{-1}\ °C^{-1}$	$cal\ g^{-1}$	$cm^2\ sec^{-1}$	$g\ cm^{-3}$	$cm^2\ sec^{-1}$	$g\ cm^{-1}\ sec^{-1}$	$g\ mol^{-1}$	$cal\ °K^{-1}\ mol^{-1}$	$g\ °K\ cal^{-1}$
-0°	$5.80 \cdot 10^{-5}$	677.0	0.206	$4.847 \cdot 10^{-6}$	0.1346	$1.718 \cdot 10^{-4}$	18.0153	1.98583	9.0721
-5°	$5.72 \cdot 10^{-5}$	677.3	.200	$3.246 \cdot 10^{-6}$	.1304	$1.692 \cdot 10^{-4}$			
-10°	$5.63 \cdot 10^{-5}$	677.5	.194	$2.139 \cdot 10^{-6}$	.1259	$1.667 \cdot 10^{-4}$			
-15°	$5.54 \cdot 10^{-5}$	677.7	.187	$1.387 \cdot 10^{-6}$	.1216	$1.641 \cdot 10^{-4}$			
-20°	$5.54 \cdot 10^{-5}$	677.9	.180	$.884 \cdot 10^{-6}$	.1173	$1.615 \cdot 10^{-4}$			

1/ Smithsonian Meteorological Tables (List 1966.)

2/ International Meteorological Tables (WMO-No. 188, TP. 94); taken as 7 percent less than table value.

Table of Variation with Pressure at -20°C

Elevation[1] (km)	P	ρ_a[2]	ν[3]	D[4]	S_o[5]
	mb	kg/m^3	$cm^2\ sec^{-1}$	$cm^2\ sec^{-1}$	$cm^{-2}\ min$
0	1000	1.4039	0.1150	0.180	0.8
1	900	1.2635	.1279	.200	1.1
2	800	1.1231	.1439	.225	1.3
	750	1.0549	.1532	.240	1.4
3	700	.9867	.1638	.257	1.46
	650	.9162	.1763	.277	1.53
4	600	.8457	.1908	.300	1.60

1/ Rough estimate from NACA standard atmosphere.

2/ Computed from virtual temperature (see Smithsonian Meteorological Tables (List 1968).)

3/ Computed from $\nu = \mu/\rho_a$.

4/ Computed from standard equation (see text).

5/ A roughly parabolic distribution drawn from Geiger (1965).

LIST OF SYMBOLS

a = $-[1/ku_* \ln(z/Z)]$ a parameter determined by the logarithmic wind profile (sec/cm)

b = major semi-axis of ellipsoid of revolution (cm)

c = minor semi-axis of ellipsoid of revolution (cm)

A = a constant

A_1 = project area of particle perpendicular to direct solar radiation (cm^2)

A_2 = projected area of particle perpendicular to reflected solar radiation from horizontal snow surface

B = a constant

C = electrostatic capacitance (shape factor for mass transfer)

C_{abs} = particle absorbing cross section (cm^2)

C_1 = computational grouping of factors depending on meteorological conditions (g/cm sec)

C_2 = fall velocity shape factor (2440 sec^{-1} $\leq C_2 \leq$ 3880 sec^{-1})

C_p = specific heat of air [$\approx$0.240 cal/°K (g dry air)]

C_v = coefficient of compressibility

D = diffusivity of water vapor in air (cm^2/sec)

(dm/dt) = sublimation rate for a single particle (g/sec)

$(dm/dt)_x$ = sublimation rate for a particle with diameter x (g/sec)

$(dM/dt)_C$ = total sublimation rate for an arbitrary column of air and snow (g/sec)

(dM/dt) = sublimation rate of mass M of snow in an arbitrary volume (g/sec)

$(dM/dt)_{zx}$ = sublimation rate of the mass of particles M_x with diameters x to (x + dx) in an arbitrary volume at height z (g/sec)

$(dM/dt)_z$ = sublimation rate for the mass of snow in arbitrary volume at height z, (g/sec)

e = 2.718 . . .

E = vertical flux of water vapor (g/cm sec)

E_{abs} = particle absorbing efficiency

$f(x)$ = general particle size distribution

F = 1/2 (Nu), wind factor

F_m = ventilation factor for mass transfer

F_h = ventilation factor for heat transfer

G = particle geometrical cross section (cm^2)

H = heat flux (cal/cm^{2}sec)

k = von Karman's constant, 0.4

K = thermal conductivity of air (cal/cm sec °K)

K_h = turbulent heat transfer coefficient (cm^2/sec)

K_m = eddy viscosity (cm^2/sec)

K_w = eddy diffusivity for water vapor (cm^2/sec)

L_s = latent heat of sublimation (cal/g)

m = particle mass (g)

$\mathcal{M}$ = molecular weight of water (g/mol)

M = mass of snow in an arbitrary volume (g/volume)

M_x = mass of snow with particle diameters, x to (x + dx) in an arbitrary volume (g/volume)

M_{zx} = mass of particles with diameter x to (x + dx) in an arbitrary volume at any height z (g/volume)

M_{Zx} = mass of particles with diameter x to (x + dx) in an arbitrary volume at reference height Z (g/volume)

n = exponent in diffusivity equation (1.75)

N = number of particles in an arbitrary volume

N_z = number of particles in an arbitrary volume at height z

N_Z = number of particles in an arbitrary volume at the reference height

N_{zx} = number of particles with diameter x to (x + dx) in an arbitrary volume at any height z

N_{Zx} = number of particles with diameter x to (x + dx) in an arbitrary volume at reference height Z

(Nu) = Nusselt number (2rK/D)

p = atmospheric pressure (mb)

(Pr) = Prandtl number ($\mu C_p/K$)

$\mathcal{P}$ = general function of height

Q = net rate of heat transfer by radiation (cal/sec)

$\mathcal{Q}$ = general function of height

r = radius (cm)

$\mathcal{R}$ = general function of height

R = universal gas constant (cal/mol °K)

(Re) = Reynolds number ($2rV/\nu$) or (xw/ν)

(RH) = relative humidity used throughout as the water vapor density in percent of the saturation value over ice

(Sc) = Schmidt number ($\mu/\rho_a D$)

(Sh) = Sherwood number (2rK/D)

S_o = total solar radiation flux (cal/cm^{2}sec)

t = time (sec)

T = environmental temperature (°K)

T_r = particle surface temperature (°K)

T_s = temperature of horizontal snow surface (°K)

T_v = virtual temperature (°C)

u = instantaneous air velocity (cm/sec)

$\bar{u}$ = mean horizontal wind velocity (cm/sec)

u_p = instantaneous wind velocity

u_R = $(u_p - u)$ relative velocity

u_* = friction velocity $u_* \equiv \sqrt{\tau_o/\rho_a}$ (cm/sec)

$\overline{u^2}$ = mean square horizontal velocity of air (cm^2/sec^2)

V = ventilation velocity of air relative to particle (cm/sec)

w = particle fall velocity (cm/sec)

x = nominal particle diameter [diameter of sphere with equal volume] (cm)

$\bar{x}$ = mean particle diameter

z = height measured from horizontal surface (cm)

z_o = roughness parameter (cm)

Z = reference height (cm)

α = parameter of particle size distribution

β = parameter of particle size distribution (cm)

$\Gamma(\alpha)$ = gamma function in α

ε = eddy viscosity (cm^2/sec)

ζ = particle surface albedo

λ = albedo of horizontal snow surface

μ = micron, 10^{-4} cm

μ = dynamic viscosity (g/cm sec)

ν = kinematic viscosity (cm^2/sec)

ξ = ratio of major to minor semi-axis (b/c) for prolate spheroid

Θ = adiabatic lapse rate (.01°C/m)

π = 3.14 . . .

ρ = water vapor density in the remote environment (g/cm^3)

ρ_a = density of air (g/cm^3)

ρ_p = particle density (g/cm^3)

ρ_r = water vapor density near particle surface (g/cm^3)

ρ_{sT} = saturation density of water vapor at temperature T (g/cm^3)

σ = $(\rho/\rho_{sT} - 1)$ undersaturation of the environment with respect to water vapor [(RH) - 100] = 100σ in percent

τ = turbulent shear stress (dynes/cm^2)

τ_o = surface shear stress (dynes/cm^2)

ϕ = volume fraction of snow in air

$1 - \phi$ = voidage

q = specific humidity

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Schmidt, R. A., Jr.
1972. Sublimation of wind-transported snow — a model. USDA Forest Serv. Res. Pap. RM-90, 24 p. Rocky Mountain Forest and Range Experiment Station, Fort Collins, Colorado 80521.

Sublimation from blowing snow is estimated from the balance of heat and mass transfer on a volume of air and nonuniform ice particles. Size and the distribution of sizes are important particle factors, but fragmented and abraided shapes appear to increase sublimation only slightly above the estimate for spherical particles. Humidity and temperature are the overriding environmental factors, but atmospheric pressure and solar radiation are also important. The effect of turbulence on convection around the particles does not appear important in view of the small particle sizes encountered. Rather, turbulent transfer determines the snow concentration profile, and the gradients of heat and water vapor necessary to balance the sublimation process.

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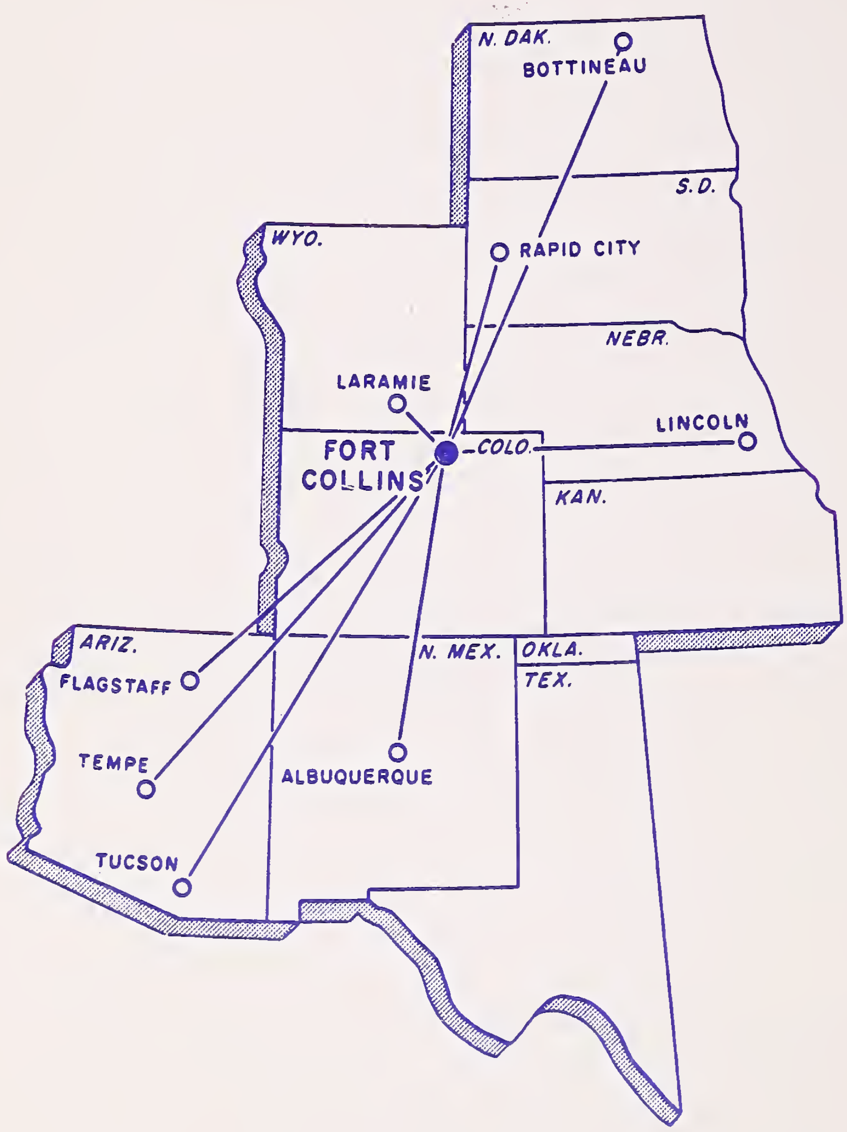
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